Content

1. Experimental Setup

Using the setup shown in FIGURE 1 various optical phenomena like refraction, reflection, and total internal reflection will be investigated. With the laser module, mounted on the moving rail, the laser light can be directed on different samples. The set of samples includes a plane-parallel plate, transmission cavity, prism, and a moving table with small prisms and lenses. The traces of the laser beam can be seen very well in the set up. The beam will be projected on a screen and small beam terminator.

The different phenomena will be investigated using special templates prepared for each test, on which all beam trajectories have to be drawn.

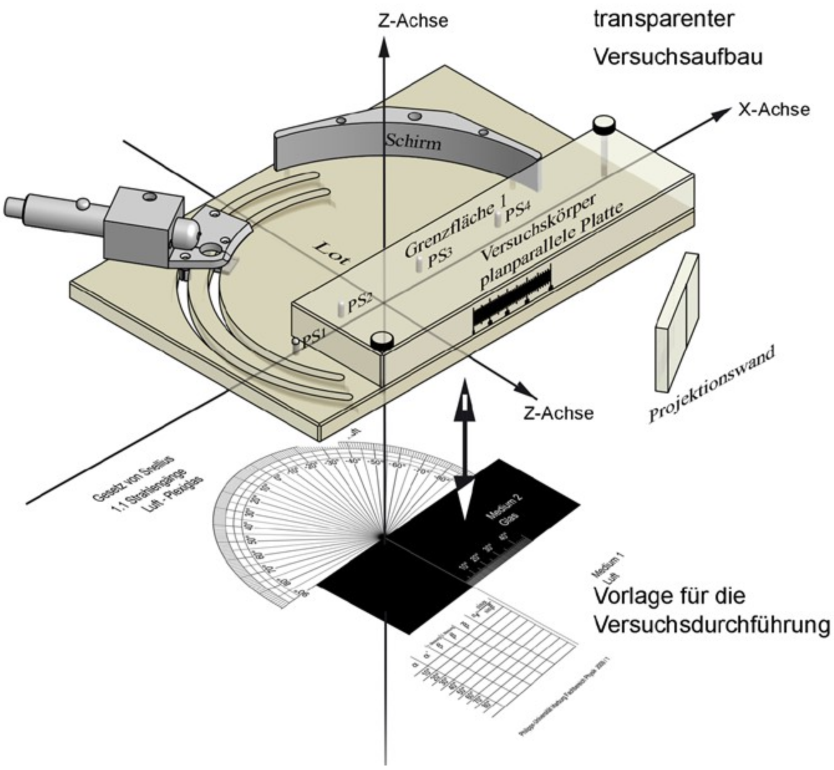

FIGURE 1: Setup for transparency test.

2. Theory

Light exists as long as time, showing images from the beginning of the universe. The importance of light became even higher with the invention of the laser: from this time on light was not only information carrier, but became a base of key technologies in 21st century.

Employment of lasers accelerated industrial processing in welding, cutting, drilling, and profiling and at the same time increased the precision of work. In medicine the laser technologies are used practically in all disciplines every day, for example, in endoscopy based on a total internal reflection phenomena. The modulated optical signal transmits information with the velocity of light through transmission systems and waveguides. Further development of laser systems led to a creation of small modules covering wide spectrum range, including wavelengths visible for human eyes.

2.1. Reflection

The transition between two media splits the beam into two parts – the reflected and transmitted beam. In case of glass, the reflected beam has 4%, while the transmitted one has 96% of the power relative to the incoming beam power. The trajectory of the reflected light is described by the reflection law, and the trajectory of the transmitted light is described by a Snellius law

$$\begin{aligned} \alpha &= \alpha' = \alpha'' \\ \beta &= \beta' \end{aligned} \tag{1.1}$$

It defines the relation between the incident angle and angle of reflection (FIGURE 2).

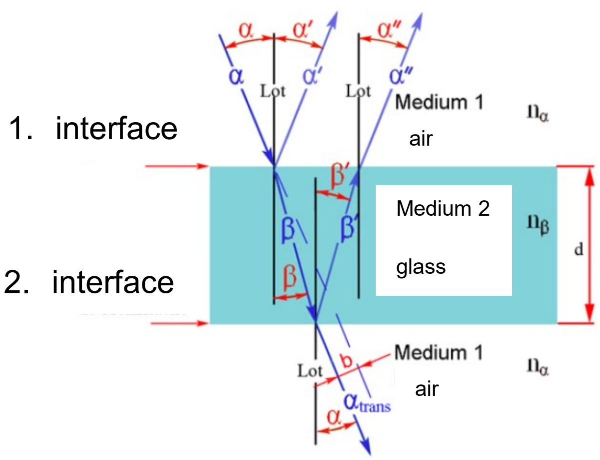

FIGURE 2: Transmitted and reflected beam trajectories.

2.2. *Snellius law*

Snellius law describes the relation between the incoming and refracted angle using n_α and n_β, which are the refractive indices of the media

$$\frac{\sin\alpha}{\sin\beta} = \frac{n_\beta}{n_\alpha} = n_{\alpha\beta} \tag{1.2}$$

We consider beam trajectories shown in FIGURE 2. The α-beam emitted by a laser is split at the interface of surface 1 into a reflected and a transmitted beam. The β-beam is transmitted through the medium 2 (here: plexiglass). On the interface of surface 2 the β-beam is split again into a transmitted beam α_{trans}, which propagates further in medium 1 (here: air) and a reflected beam β´. The beam β´ is split again on the interface of surface 1, but only beam α´´ is observable. The intensity of α´´ is almost equal to the intensity of α´. The second beam α´´ can be strongly scattered by optical elements. To minimize this effect, optical components usually have antireflection coatings.

The refractive indices n_i have subscripts (α, β, γ etc.), which denote the refractive index relative to vacuum, which is the minimum possible refractive index for any transparent media. Due to the extremely small difference between refractive indexes of vacuum and air, in literature, they are often considered as identical

$$n_{vacuum} = n_{air} = n_\alpha \tag{1.3}$$

An accuracy of angle measurements in the lab is about 0.1°. Because of that, the refractive index of air is also given only with the accuracy of two digits: n_α = 1.00.

Table 1: Various, relative refractive indices.

Optical medium	Refractive index relative to vacuum	Refractive index used in this lab
air	n_{air} = 1.00003	n_{air} = 1.00
crown glass K13	n_{K13} = 1.52238	n_{K13} = 1.52
flint glass F2	n_{F2} = 1.61990	n_{F2} = 1.62
heavy flintglass SF3	n_{SF3} = 1.73976	n_{SF3} = 1.74
diamond	$n_{diamond}$ = 2.41730	$n_{diamond}$ = 2.42

The values of the refractive index at an interface determine the trajectory at the surface. The refractive index is always greater then 1, $n_i \geq 1$, but its relative counterpart $n_{\alpha\beta} = \frac{n_\beta}{n_\alpha}$ can have any value.

Table 2: Different relative refractive indices and beam trajectories.

Relative refractive index at interface	Beam trajectory change	Example
$n_{\alpha\beta} > 1$	refractive beam get closer to perpendicular	air to glass
$n_{\alpha\beta} = 1$	no refraction	same media
$n_{\alpha\beta} < 1$	refractive beam move away from perpendicular	glass to air

2.3. Parallel shift

We consider again beam propagation entering and exiting a plane parallel plate (FIGURE 2). The beam α, incident on a plane parallel plate at the angle 0 ≠ α, propagates through this plate with a width d and angle β to a perpendicular. On the interface of surface 2, the beam exits the plate at the angle α_{trans}. There is no change in propagation direction, but rather a shift in comparison to a trajectory without the plane parallel plate. This parallel shift is easily calculated. We first calculate the angle β in the medium and transmitted angle α_{trans} into air using Snellius law twice.

$$\alpha \neq 0 \tag{1.4}$$

Calculation of β in glass

$$\sin\beta = \frac{n_\alpha}{n_\beta} \cdot \sin\alpha \tag{1.5}$$

Calculation of α_{trans} in air

$$\sin\alpha_{trans} = \frac{n_\beta}{n_\alpha} \cdot \sin\beta \tag{1.6}$$

(1.5) in (1.6):

$$\sin\alpha_{trans} = \frac{n_\beta}{n_\alpha} \cdot \frac{n_\alpha}{n_\beta} \cdot \sin\alpha \tag{1.7}$$

yields

$$\alpha = \alpha_{trans}, \tag{1.8}$$

and the parallel shift b is given by the following equation

$$b = d\,\frac{\sin(\alpha - \beta)}{\cos\beta}. \tag{1.9}$$

2.4. Total internal reflection

We investigate this effect at the transition between water and air (FIGURE 3). We denote air as medium 1 and water as a medium 3. A beam γ is split into a refracted beam α and reflected one γ´ at the interface between water and air. We increase the angle γ and observe the change of the transmitted angle α. At some particular angle γ the refracted beam α vanishes and, only the reflected beam γ´ is left.

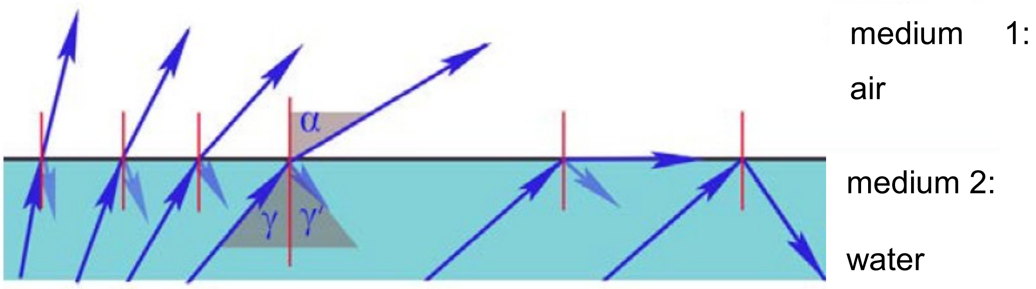

FIGURE 3: Transition of light from dense to less dense medium.

The product

$$\frac{n_\gamma}{n_\alpha}\sin\gamma > 1 \tag{1.10}$$

with

$$\sin\alpha = \frac{n_\gamma}{n_\alpha}\sin\gamma \tag{1.11}$$

has no solution, because the function y = sin(x) is not defined for $|y| > 1$. Therefore, we do not observe the beam anymore, which leaves the water on the top surface (FIGURE 3). This angle is denoted as the angle of total internal reflection, and the phenomenon is called total internal reflection (TIR). The angle of total internal reflection is given by

$$\gamma_{TIR} = \arcsin \frac{n_\alpha}{n_\gamma} \tag{1.12}$$

The total internal reflection phenomenon is used in medical endoscopes and optical waveguides.

2.5. Beam propagation through prism

In case of the propagation of a beam through a prism, the incoming beam will be refracted twice towards the direction of basis of the prism. The total refraction δ is calculated using the following equations

$$\delta = (\alpha_1 + \alpha_2) - (\beta_1 + \beta_2) \tag{1.13}$$

$$\sin \beta_1 = \frac{n_\alpha}{n_\beta} \sin \alpha_1 \tag{1.14}$$

$$\delta = (\alpha_1 + \alpha_2) - \gamma_1 \tag{1.15}$$

$$\beta_2 = \gamma_1 - \beta_1 \tag{1.16}$$

$$\gamma_1 = \beta_1 + \beta_2 \tag{1.17}$$

$$\sin \alpha_2 = \frac{n_\beta}{n_\alpha} \sin \beta_2 \tag{1.18}$$

If the angle α_1 is less than α'_1, total internal reflection takes place on the other side of the prism.

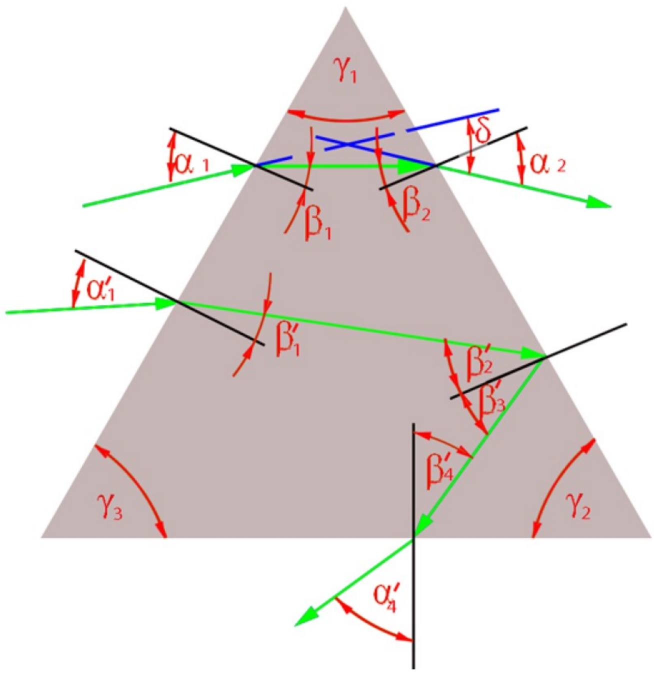

FIGURE 4: Refraction and reflection in a prism.

Angle of minimum deviation between the incident and transmitted beams appears when the beam propagates parallel to the base inside of the prism $|\alpha_1| = |\alpha_2|$. In this case, the refractive index of the prism can be calculated according to the expression

$$n_\beta(\lambda) = \frac{\sin\left(\frac{\alpha_{\min}}{2} + \frac{\gamma}{2}\right)}{\sin\frac{\gamma}{2}}, \tag{1.19}$$

where γ is the prism angle.

2.6. *Prism dispersion*

Dispersion of a prism exhibits itself as a wavelength dependence of the refraction angles of the beams. For example, two beams with different wavelengths (red and green) are refracted by a different angle (FIGURE 5).

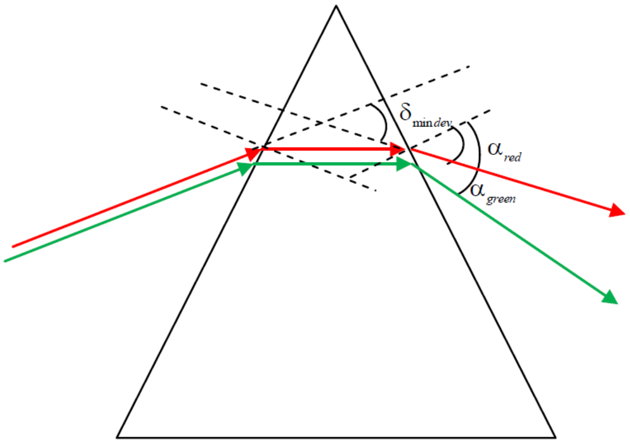

FIGURE 5: Diffraction in a prism.

Dispersion of a prism is characterized by:

$$\text{Dispersion} \approx \frac{\partial \alpha}{\partial \lambda} \approx \frac{\alpha_{red} - \alpha_{green}}{\lambda_{red} - \lambda_{green}} \tag{1.20}$$

It can be shown that the dispersion has maximum value for the beam propagating parallel to the base of the prism, which corresponds to the angle of minimum inclination, e.g. $\delta_{\min,dev} = \alpha_1 - \alpha_2$ (FIGURE 5) between the continuations (dashed lines) of the incident and refracted beams.

2.7. Grating dispersion

If a plane wave is diffracted on a grating (FIGURE 6), each point of the grating creates secondary waves, which interfere with different phase shifts resulting in different diffraction orders. A grating consists of many thin slits with period d. A grating is characterized by a number of slits N per mm, having typical values of 100mm^{-1}, 300 mm^{-1} or 600 mm^{-1}.

Each wavelength is diffracted at the angle Θ, which is calculated by:

$$\sin \Theta = \frac{k\lambda}{d}, \; k = 0, \pm 1, \pm 2, \ldots \tag{1.21}$$

This equation shows that the longer wavelength (red) is diffracted by a larger angle compared to the shorter ones (green). Due to constructive interference, two symmetric sets of different orders appear on the screen.

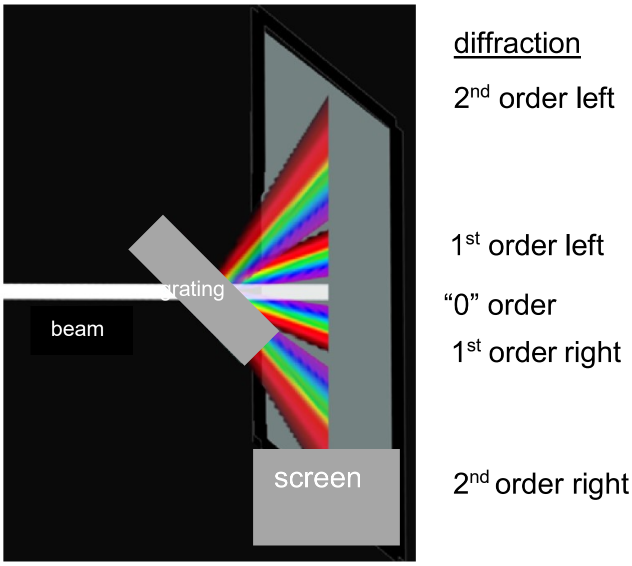

FIGURE 6: Grating generates different diffraction orders, each of them containing a diffracted spectrum.

1.1. Light as electromagnetic wave

We consider an electromagnetic wave. This wave consists of electric $\vec{E}$ and magnetic $\vec{H}$ fields, orthogonal to each other and propagating along the $\vec{x}$ - axis. The fundamental properties of the wave propagation are described by the Maxwell equations.

FIGURE 7: An electromagnetic wave $\vec{E} \perp \vec{H} \perp \vec{x}$ (left); electromagnetic waves polarized in incident plane $\vec{E}_z$ and perpendicular to the incident plane $\vec{E}_y$ (right).

When polarization effects are discussed, only the electric field is considered. Here we consider the propagation in $\vec{x}$ - direction and projections of the field polarization on the incident plane $\overline{xz}$ and perpendicular to the incident plane.

<u>Brewster angle (David Brewster, 1781 – 1868)</u>

The polarization properties of an electromagnetic wave can be shown at reflection under Brewster angle, where one of the polarizations experiences no reflection. The electric field consists of parallel E_{ep} and perpendicular E_{es} components the incident plane fields. One can calculate reflection and transmission coefficients for the both waves (transmission is not considered hereafter).

$$r_s = \frac{E_{rs}}{E_{es}} \tag{1.22}$$

and

$$r_p = \frac{E_{rp}}{E_{ep}} \tag{1.23}$$

and calculate intensity reflection for both polarizations

$$R_s = r_s^2 \tag{1.24}$$

$$R_p = r_p^2 \tag{1.25}$$

Under the Brewster angle, only one polarization E_s is reflected ($R_s \neq 0$), while the other one is completely transmitted ($R_p = 0$).

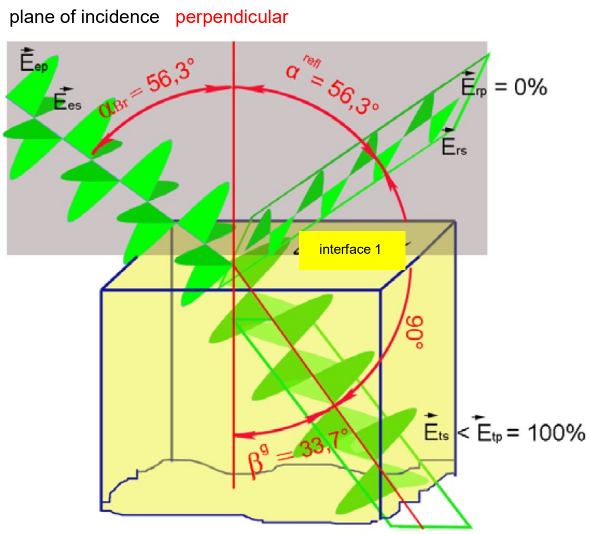

FIGURE 8: Reflection of an incident electromagnetic wave for parallel E_p and perpendicular E_s to the incident plane polarizations under Brewster angle: only one polarization E_{es} is reflected ($R_s \neq 0$), while the other one is fully transmitted ($R_p = 0$).

The Brewster angle α_{Br} is a special solution of the Fresnel equations, describing reflection and transmission at boundaries.

<u>Reflection for polarization perpendicular to the incident plane:</u>

$$R_s = r_s^2 = \frac{\sin^2(\alpha - \beta)}{\sin^2(\alpha + \beta)} \tag{1.26}$$

<u>Reflection for polarization parallel to the incident plane:</u>

$$R_p = r_p^2 = \frac{\tan^2(\alpha - \beta)}{\tan^2(\alpha + \beta)} \tag{1.27}$$

The Brewster angle is given by the condition $R_p = 0$:

$$\alpha_{Br} + \beta^g \approx 90°, \tag{1.28}$$

when the reflected and refracted beam are perpendicular to each other. The transmission of the wave with the polarization parallel to the incident plane for this angle is 100%.

<u>Brewster angle calculation</u>

Applying Snellius law, one can easily calculate the Brewster angle

$$\frac{\sin\alpha_{Br}}{\sin(90° - \alpha_{Br})} = \frac{n_\beta}{n_\alpha} \tag{1.29}$$

$$\frac{\sin\alpha_{Br}}{\cos\alpha_{Br}} = \frac{n_\beta}{n_\alpha} \tag{1.30}$$

Then, the so-called “external” Brewster angle from less dense to dense media is

$$\tan\alpha_{Br} = \frac{n_\beta}{n_\alpha} \tag{1.31}$$

In case of transition from air:

$$n_\beta = \tan\alpha_{Br} \tag{1.32}$$

The angle dependence of the intensity reflection coefficients R_s and R_p for both polarizations are shown in FIGURE 9.

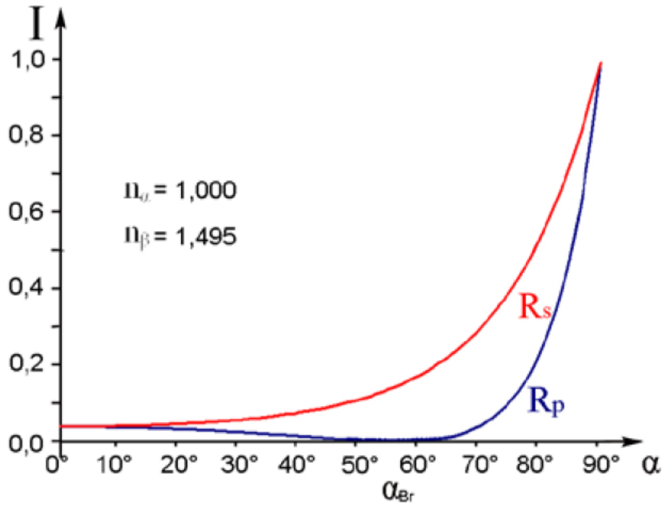

FIGURE 9: Intensity reflection coefficients for the both polarizations in case of transition from air to plexiglass.

2.8. Birefringence and optical activity

Birefringent crystals have different refractive indices for different propagation direction. Here, we deal only with the single axis birefringent crystals, which have to refractive indices, namely n_o (ordinary) and n_{eo} (extraordinary).

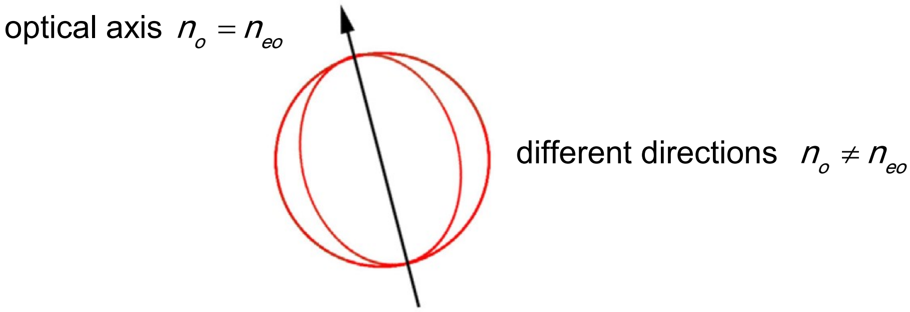

FIGURE 10: Optical axis of a birefringent crystal.

<u>Transmission in birefringent crystal cut parallel to optical axis</u>

A plane wave, travelling through a birefringent crystal which is cut parallel to an optical axis, shows a rotation of the polarization. This effect can be detected using two crossed polarizers. Suppose, the incoming, unpolarized wave front consists of two wave fronts with two mutually perpendicular polarization states E_{ep} and E_{es}. At the polarizer P (FIGURE 11) the wave front with the polarization state E_{es} will be absorbed. The wave front after the polarizer P is linearly polarized. In case of the second polarizer is placed perpendicular to the first one, the second wave front E_{ep}, and hence the transmission, is completely blocked.

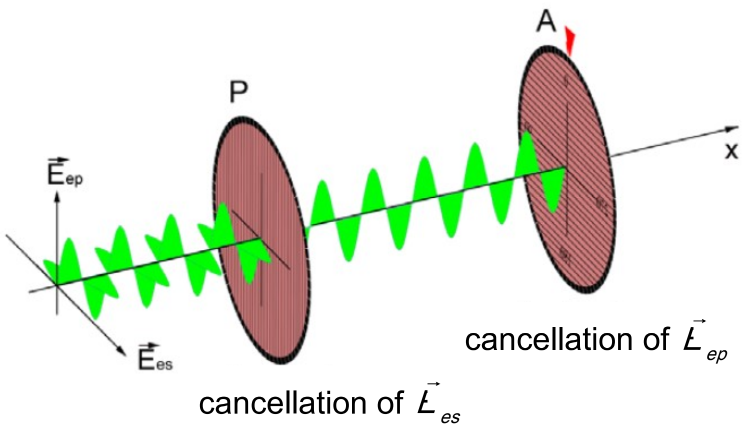

FIGURE 11: Transmission is blocked by two perpendicular aligned polarizers.

The birefringent crystal is placed between the polarizers with optical axes parallel to the propagation direction. After the first polarizer, the linearly polarized wave front propagates through the crystal. The polarization plane is rotated, and the second polarizer does not block the light completely. After angle adjustment of the second polarizer the rotation angle can be measured. The rotation angle depends on the wavelength and thickness of the crystal. The optical activity is shown here with SiO_2; right- and left rotating crystals exist, depending on the crystal structure.

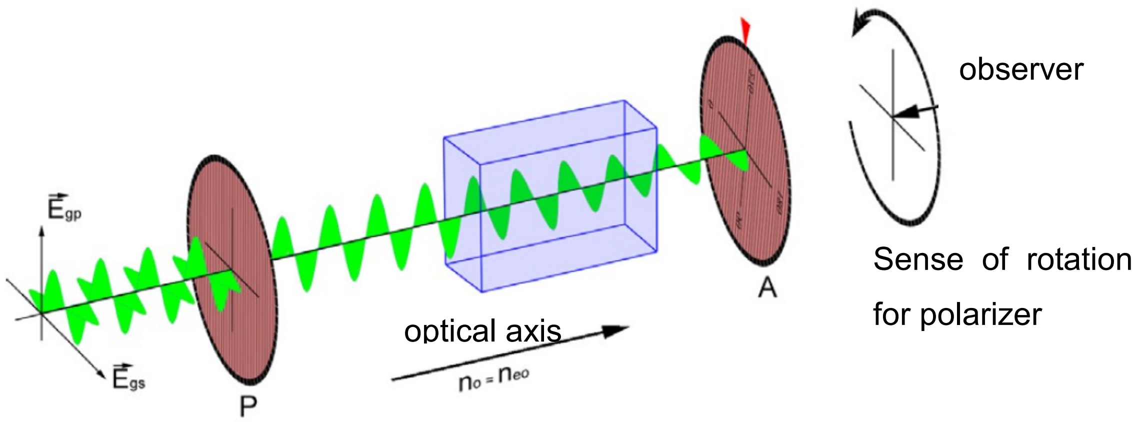

FIGURE 12: Linearly polarized wave propagates parallel to optical axis of a birefringent crystal. The polarization plane rotates during the propagation through the crystal.