Strong fields in laser plasmas
- an introduction to high intensity laser plasma interaction -
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1 Laser acceleration experiments
   Experimental techniques & measurement of strong fields

2 Electrons kinematics
   and laser absorption

3 Principles of laser – particle – acceleration
   - overview of some subjects concerning
     actual application of high intensity lasers
   - few basic considerations (not a consistent theoretical approach)
   - overview of experiments and techniques

Centers in Germany: FSU, GSI, HHU, LMU-MPQ, MBI
   european activities cf. e.g.: www.extreme-light-infrastructure.eu

references
at the end

lecture is a
compilation
out of books,
PhD-thesis,
articles,
(Don’t blame me
for footnotes 😊 )
Grading of ´´strong´´, ´´high´´ ⇔ ´´relativistic´´

particle gains energy in field => kinematics starts to become relativistic
say:
\[ T = (\gamma - 1) m_0 c^2 = m_0 c^2 \quad (\gamma = 2) \]
\[ \gamma = (1 - \beta^2)^{-1/2} \quad \beta = v/c \]

electron in oscillatory E-field

\[ m_e \ddot{x} = e E_0 \cos (\omega t) \]
integration
\[ v(t) = e E_0 / (e \omega) \sin (\omega t) \]

max.kinetic energy (non.rel.) = energy equivalent of rest mass

\[ m_e / 2 v^2(t) = e^2 E_0^2 / (2 m_e e^2 \omega^2) \sin^2 (\omega t) = m_e c^2 \]

maximum energy: \[ e^2 E_0^2 / (2 m_e e^2 \omega^2) = U_{\text{max}} \]

cycle T (\omega = 2\pi/\omega) averaged energy

\[ \int_0^T v(t) \, dt = e^2 E_0^2 / (4 m_e e^2 \omega^2) = U_p \]
calculation concerning max. kinetic energy

\[ E_0 = \sqrt{2} m_e \omega c / e \]

relate \( \omega = 2\pi f = 2\pi c / \lambda \) (\( \lambda = 800 \text{ nm} \))

\[ E_0 = 5.7 \times 10^{10} \text{ V/cm} \]

call \( e E_0 / (m_e \omega c) = a_0 \) dimensionless (normalized) field amplitude

\( a_0 > 1 \) region of relativistic kinematics

\( a_0 = 1 \) boundary \( a_0 = v_{osc} / c = 1 \)

such high field strength can be produced with em-fields of lasers
if one associates \( E_0 \) with a strong em-field of an intensity \( I \)

one needs \( I \approx 4.3 \times 10^{18} \text{ W/cm}^2 \) or \( I \approx 2.1 \times 10^{18} \text{ W/cm}^2 \) (in case \( a_0 = 1 \))

\[ E_0 [\text{ V/m}] \approx 27.4 \sqrt{I} [\text{ W/cm}^2] \]

=> have to look to electron kinematics in “strong” em-fields

cf. atomic units
field strength ~ \( 5 \times 10^9 \text{ V/cm} \)

cf. below
additional association
with \( a_0 \)

em-field
wave equation
solution
Poynting vector

M. Schnürer: Strong fields in laser plasmas 2 – Electron kinematics and laser absorption 1/15/2012 3
Single electron interaction with an em – wave

Maxwell equations
\[ \text{rot } \mathbf{E} = - \frac{\partial \mathbf{B}}{\partial t} \quad \text{rot } \mathbf{B} = \mu_0 \varepsilon_0 \mathbf{j} + \frac{\partial \mathbf{E}}{\partial t} \quad \text{div } \mathbf{E} = \rho / \varepsilon_0 \quad \text{div } \mathbf{B} = 0 \]

Introduction of vector potential \( \mathbf{A} \)
\[ \mathbf{E} = - \frac{\partial \mathbf{A}}{\partial t} \quad \mathbf{B} = \text{rot } \mathbf{A} \]

Can derive wave equation, solution e.g.
A propagating wave in \( x \) – direction expressed by
\[ \mathbf{A} = (0, \delta A_0 \cos (\phi), (1 - \delta^2)^{1/2} A_0 \sin (\phi)) \]
\[ \phi = \omega t - k x \]
\[ \delta = \{ \pm 1, 0 \} \text{ linear polarization} \]
\[ \delta = \{ \pm 1/Sqrt(2) \} \text{ circular polarization} \]

Or e.g.
\[ \mathbf{E}_{\text{lin}} = E_0 \sin (\omega t - k x) \mathbf{e}_y \]

One can derive an energy density of the em – field \( w_{\text{em}} \)
And can associate an energy current density \( w_{\text{em}} \mathbf{c} = \mathbf{S} \) (Pointing vector)

Which gives the relation to the intensity \( I \) (cycle averaged)
\[ I = |\mathbf{S}| = \frac{1}{2} \varepsilon_0 c E_0^2 \]
useful relation

\[ a_0^2 = I \left[ \text{W/cm}^2 \right] \lambda^2 \left[ \text{\mu m}^2 \right] \times 0.73 \times 10^{-18} \]

plane wave -> pulse

temporal function of field amplitude – envelope \( E_{\text{env}}(t) \)
e.g. field varies

\[ E(t) = E_{\text{env}}(t) \cos(\omega t - \varphi t) \]
Gauss: \( E_{\text{env}}(t) = \exp \left( -\frac{t}{\tau_p} \right)^2 \)

plot example:

FWHM ~ 8 cycles ~ 22 fs
in case of a 800 nm Ti:Sapph.laser
Electron - field interaction
- plane wave

Lorentz equation:
\[ \frac{dp}{dt} = \frac{d}{dt} (\gamma m_e v) = -e (E + v \times B) \]

non.-rel. case and \( v \times B = 0 \) cf. previous solution of relativistic case:
trajectories:
\[
\begin{align*}
x &= \frac{c a_0^2}{4\omega} (\Phi + \frac{2\delta^2 - 1}{2 \sin(2\Phi)}) \\
y &= \delta \frac{c a_0}{\omega \sin(\Phi)} \\
z &= -\left(1 - \delta^2\right)^{1/2}/\omega \cos(\omega)
\end{align*}
\]

example plots:

cf. references
P.Gibbon
2005
PhD-thesis
Sokollik 2009
Steinke 2010
Figure 1.1: A - Electron trajectory caused by a infinite plane wave \( (\alpha_0 = 2) \) (laboratory frame). B-D - Electron trajectories for a pulse duration of 15 fs with same maximum intensity.

Figure 1.1: Electron trajectory in case of (a) linear and (b) circular polarization of the incident laser pulse, propagating in \( x \)-direction with a pulse duration of \( \tau = 15 \text{ fs} \) and \( \alpha_0 = 5 \). The net energy gain of the electron is zero in both cases, it is only displaced in laser pulse propagation direction.
need high intensity, high field strength

plane wave -> focused wave -> spatial field distribution

Example plot: focused gaussian beam with $w_0 = 5$ beam waist parameter

$\rightarrow$ consequences of field gradients
Ponderomotive force

- Electron in an em – wave
  (propagation along x, frequency ω, wavenumber k)
  non.rel. equations of motion:

\[
\frac{dv_y}{dt} = \frac{e E_0}{m_e} \cos (\omega t - kx)
\]

\[
\frac{dv_x}{dt} = v_y e B / m_e \cos (\omega t - kx)
\]

- investigation: how does the spatial change of the field amplitude act on electron movement ( important situation -> field distribution in a focus)

- look to a principle effect ( simplification – non-relativistic treatment and influence of B-field is omitted v_y B -> 0, 2nd equation cancelled

-> integration first equation:

\[
v_y = \frac{e E_0}{(m_e \omega)} \sin (\omega t - kx)
\]

\[
y = - \frac{v_{osc}}{\omega} \cos (\omega t - kx)
\]

\[
v_{osc} = \frac{e E_0}{(m_e \omega)}
\]
-introduce inhomogeneous field,
description with a 2\textsuperscript{nd} order term concerning $E_0$

$$E_0 \rightarrow E_0 + y \frac{dE_0}{dy}$$

$$\rightarrow \quad \frac{dv_y}{dt} = \frac{e}{m_e} \left( E_0 + y \frac{dE_0}{dy} \right) \cos \left( \omega t - kx \right) \quad \text{with} \quad y = \ldots$$

$$\rightarrow \quad \frac{dv_y}{dt} = \frac{e}{me} E_0 \cos \left( \omega t - kx \right) - \frac{v_{osc}}{\omega} \frac{e}{m_e} \frac{dE_0}{dy} \cos^2 \left( \omega t - kx \right)$$

! look now to time - averaged effect $\int_0^T \frac{dv_y}{dt} \ dt$!

$$\int_0^T \cos(.) \ dt = 0 \quad \int_0^T \cos^2(.) \ dt = \frac{1}{2}$$

and $v_{osc}/\omega \frac{e}{m_e} \frac{dE_0}{dy} = e^2/(m_e \omega^2) E_0 \frac{dE_0}{dy}$

$$= \frac{1}{2} \left( e/(m_e \omega) \right)^2 \frac{1}{2} \frac{d}{dy} (E_0^2)$$

can associate

$$\rightarrow \quad F_p = - \frac{1}{4} m_e \left( e/(m_e \omega) \right)^2 \frac{d}{dy} (E_0^2)$$

the ponderomotive force
(act on a charge in an oscillating em-field)
Ponderomotive potential – useful relations

- can associate $F_p$ with the ponderomotive potential:

$$U_p \sim \int F_p \, dy \sim \frac{1}{4} \varepsilon_0 E_0^2/(m_e \omega^2) \equiv \text{cycle averaged oscillation energy}$$

- general relativistic treatment gives same relation!

$$(U_p \leftrightarrow I_L - \text{relation})$$

- with some plasma terms (cf. next considerations)

$$U_e = \frac{1}{2} \varepsilon_0 E_0^2 \text{ em-field energy density}$$

$$\omega_p = n_e e^2 / \varepsilon_0 m_e \quad n_c = \omega_p^2 \varepsilon_0 m_e / e^2$$

one can write force per unit volume

$$f = n_e m_e \frac{dv_y}{dt} = - \frac{1}{2} (\omega_p / \omega) \frac{d(\frac{1}{2} \varepsilon_0 E_0^2)}{dy} = - n_e / n_c \frac{dU_e}{dy}$$

or

$$f = - \frac{1}{2} n_e / n_c \nabla U_e = - \frac{1}{4} n_e m_e \nabla v_{osc}^2 \text{ in comparison}$$

to thermal pressure $\nabla p_{therm} = \nabla (n_e m_e v_e^2)$

ponderomotive. exceeds thermal. for $> 10^{15} \text{ W/cm}^2$
Plasma – definition – examples

**plasma** term introduced by Langmuir in 1923, investigation of electrical discharges means – something shaped – jelly like
Electromagnetic wave propagation in a plasma

MWE in a medium

-> wave equation

$$\Delta \mathbf{E} - \text{grad} \frac{q}{\varepsilon_0} - \mu_0 \frac{\delta}{\delta t} (\mathbf{j}) - 1/c^2 \frac{\delta^2}{\delta t^2} \mathbf{E} = 0 \quad c^2 = 1/(\varepsilon_0 \mu_0)$$

≡ 0 – plasma: quasi-neutrality of charge

$$\mathbf{j} = e (n_i \mathbf{v}_i - n_e \mathbf{v}_e)$$ consider only electrons because $$m_i >> m_e$$ and exclude $$v_e$$ gradients perpendicular to propagation

thus

$$\frac{\delta}{\delta t} (\mathbf{j}) = e n_e \frac{\delta}{\delta t} (\mathbf{v}_e) = e^2 n_e/m_e \mathbf{E}$$

≡ $$e \mathbf{E}/m_e$$

delivers ($$E_y$$ only)

$$\frac{\delta^2}{\delta x^2} E_y - \omega_p^2/c^2 = \frac{1}{2} \frac{\delta^2}{\delta t^2} \mathbf{E} = 0$$

with

$$\omega_p^2 = n_e e^2/(\varepsilon_0 m_e)$$ the plasma frequency
because wave equation has solution in form of

\[ E_y = E_0 \exp(-i (kx - \omega t)) \]

if

\[ k^2 = \frac{1}{c^2} (\omega^2 - \omega_p^2) \]  \hspace{1cm} \text{(dispersion relation)}

important consequence:

\( \omega < \omega_p \) -> no wave propagation,
only field penetration with exponential decay  \( \rightarrow \) skin depth ( \( \sim \frac{c}{\omega} \))

for \( \omega = \omega_p \)  \( \rightarrow \) \( n_e = n_c \) defines the critical density

\[ n_c = \frac{\omega^2 \varepsilon_0 m_e}{e^2} \]

example:

800 nm laser ,
\( m_e = 512 \) keV , \( e = 1.6 \times 10^{-19} \) As, \( \varepsilon_0 = 8.85 \times 10^{-12} \) As/Vm

\( n_c \sim 1.7 \times 10^{21} \) cm\(^{-3} \)
Debye length

Plasma: quasi neutrality as a whole but micro – fluctuation possible

Scale of possible charge fluctuations?
-> \( \lambda_D \) if 
electrostatic attraction force (potential, pressure)
\[ \equiv \] thermodynamic force (potential, pressure)

intuitive estimation:

\[
N_e \frac{e^2}{\varepsilon_0 \lambda_D} = k_B T_e \quad n_e = N_e / \lambda_D^3
\]

\[ \Rightarrow \lambda_D = (\varepsilon_0 k_B T_e / (n_e e^2))^{1/2} \]

element:

Exact treatment requires solution of Poisson equation which gives for the electrostatic potential:

\[
U_{el} \sim Z e/r \exp(-r/\lambda_D)
\]
Laser absorption in plasmas

⇒ collisional absorption

treatment:

ion – electron collision frequency $v_{ei}$

introduce in dispersion relation

⇒ $\omega_L^2 = \omega_p^2 \left( 1 - i \frac{v_{ei}}{\omega_L} \right) k^2 c^2$

\[
gives \quad k = k_{\text{real}} + i k_{\text{im}} \]

for a complex wave describes the term $\exp(-k_{\text{im}} x)$ -> absorption

can write if $\omega_L >> v_{ei}$

\[
k_{\text{im}} = v_{ei} \frac{\omega_p^2}{(2 \ n_c \omega_L)} = v_{ei} n_e / (2 c \ n_c (1 - n_e / n_c)^{1/2}) \]

use expression

\[
v_{ei} = e^4 Z_{av} n_e \ln \Lambda / (3 (2 \pi)^{3/2} \varepsilon_0^2 m_e^{1/2} (k_B T_e)^{3/2}) \]

\[
v_{ei} = 3 \times 10^{-6} Z_{av} n_e [\text{cm}^{-3}] \ln \Lambda / (T_e [\text{eV}])^{3/2} \text{ s}^{-1} \]
Coulomb logarithm \( \ln \Lambda \)

\[
\ln \Lambda = \ln \left( \frac{b_{\text{min}}}{b_{\text{max}}} \right) \quad \text{is ratio of impact parameter}
\]

schematic picture

can associate  

\( b_{\text{max}} \rightarrow \lambda_D \)

\[ b_{\text{min}} \rightarrow \text{coulomb energy } Z e^2 / (4 \pi \varepsilon_0 b_{\text{min}}) \]

= thermal energy \( 3/2 k_B T_e \)

gives approximation

\[
\ln \Lambda = 22.8 + 0.5 \ln \left( \frac{T_e^3 \text{[eV]}}{(Z_{\text{av}}^2 n_e \text{[cm}^{-3}])} \right)
\]

and for the local absorption \( k_{\text{im}}(\nu_{ei}) \)

\[
k_{\text{im}} \sim n_e^2 Z_{\text{av}} / (T_e^{3/2} (1 - n_e/n_c)^{1/2})
\]
Knowledge of $v_{ei}$ gives also access to dielectric constant (and $n_{pl}$ – refraction coefficient of plasma)

$$\varepsilon = 1 - \frac{n_e}{n_c} \frac{1}{1 + i \frac{v_{ei}}{\omega_L}}$$

Have to determine the absorption coefficient globally i.e. laser produced plasmas have density variations to consider

example: exponential density gradient

$$n_e = n_c \exp \left( - \frac{x}{L} \right) , \text{ case } L >> \lambda_D$$

absorption s-polarized light

$$A_{L,s} = 1 - \exp \left( - 8 \frac{v_{ei}(n_c)}{c} \frac{L}{(3 \cos^3(\theta))} \right)$$

is a slowly varying function with $\theta$ - incidence angle of laser

Kruer
p-polariced
cf. Gibbon

Homework: Why does absorption increase with higher laser frequency?
⇒ equipartition times

timescale -> particle in equilibrium having Maxwell distribution

t_{ee} = 0.29 (m_e c^2)^{1/2} T_e^{3/2} / (n_e c^4 \ln \Lambda)

= 0.33 (T_e[eV]/100)^{3/2} 10^{21} / (\ln (\Lambda) n_e [cm^3]) \text{ ps}

relation t_{ee} t_{ei} t_{ii}

e.g. t_{ei} \sim 1000 A/Z_{av}^2 t_{ee}

Why do we have decreasing collisional absorption
with increasing laser intensity I_L?

cross section e – i collisions: \sigma_{ei} = 4 \pi Z_{av} e^4 / (m_e^2 v_{eff}^4)

v_{eff} = (v_e^2 + v_{quiver}^2)^{1/2}

thus v_{ei} decreases and

k_{eff}(I_L) = k_{im} (1 + I_L(3/(2 c n_c k_B T_e)))^{-1}
When does collisionless absorption “overtake” collisional absorption?

can estimate with “energy” comparison of the electron movement
or “field” oscillation * coll.rate  versus “thermal” movement * coll.rate

follows:

\[ \frac{v_{\text{quiver}}^2}{v_{ei}} > \frac{Z_{av}}{v_{ei}} \frac{v_{e\text{therm}}^2}{v_{ei}} \]

if

\[ Z_{av}^{-1} \left( \frac{v_{\text{quiver}}^2}{v_{e\text{therm}}^2} \right) = \frac{I_L}{(Z_{av} c n_c k_B T_e)} = 2.1 \times 10^{-13} \frac{I_L \text{[W/cm}^2\text{]} \lambda^2 \text{[\mu m]}}{(Z_{av} T_e \text{[eV]})} \]
Collisional absorption processes

Resonance absorption

Situation:
P-polarized wave incident at angle $\theta$
target surface with plasma and density gradient $L$

$$n_e = n_c \frac{x}{L}$$

scheme

functional dependence:

$$f_a = \frac{I_{abs}}{I_L} = 36 \xi^2 \frac{Ai(\xi)}{(d Ai(\xi)/d\xi)}$$

$$\xi = (k L)^{1/3} \sin(\theta)$$

plot

principle:
resonant excitation of plasma waves with $\omega_p = \omega_L$
at $n_e = n_c$.
“tunneled” fraction of laser light to nc position
Brunel absorption
cf. PRL 1987

situation:
electron excursion length
\( x_p \sim \frac{v_{osc}}{\omega_L} > L \) (scale length of density gradient)
resonance condition at \( n_c \) breaks down
but
electron acceleration into vacuum (half cycle)
and acceleration back into plasma
\( \Rightarrow \) electrons reach region
\( \Rightarrow \) where \( em \) – wave is already strongly damped (\( n_e > n_c \))
\( \Rightarrow \) electrons “leave” \( em \) – field
\( \Rightarrow \) “acquire” kinetic energy

scheme for two cases \( a_0 << 1 / 1 << a_0 \)

simple Brunel model
triggered a lot of refined studies
Relativistic $j \times B$ heating ⇔ ponderomotive acceleration

if $a_0 > 1$

B – field contributes to ponderomotive force $F_p$
(term for ponderomotive force is similar as deduced above)

2 effects
$F_p$ pushes electrons out of the focal region
electrons gain energy

$$E_{\text{kin}} = U_p = m_e c^2 (\gamma - 1) = m_e c^2 ((1 + a_0^2)^{1/2} - 1)$$

force in x – direction (em – wave propagation)
$$F_x = - m_e / 4 \delta^2 / \delta x^2 (v_{\text{osc}}) (1 - \cos^2(\omega t))$$

pond. term oscillation in x
due to $j \times B$
(electron trajectory)
leads to “heating”

effects are very close to action of light pressure
⇒ plasma hole boring
⇒ cf. lecture 2: particle acceleration
Anomalous skin effect

for $n_e > n_c$ decaying E-field
scale is skindepth

$$l_s = c/\omega_p$$

(example: 800 nm laser $\omega_L$ 2.35 $10^{15}$ s$^{-1}$, $n_e = 100$ $n_c$, thus $\omega_p = 10 \omega_L$
and $l_s \sim 13$ nm , or lower if layer is highly ionized)

anomalous means:
mean free path between collisions $> l_s$
or
$$v_{th} / v_{ei} \sim l_c > l_s \quad v_{th} = (k_B T_e / m_e)^{1/2}$$
description of extended layer $l_c$ (the anomalous skin layer)
in which absorption takes place
$$l_c \sim (v_{th} c^2 / (\omega \omega_p^2))^{1/3}$$
and relative absorption
$$\eta_{asa} \sim (k_B T_e/ (511 \text{ keV})^{1/6} (n_e/n_c)^{1/3}$$
gives e.g. for $n_e = 100$ $n_c$ and $I_L \sim 10^{19}$ W/cm$^{-2}$
about 5% absorption

Books
Shalom Elizier
*The interaction of high-power lasers with plasmas*
IoP Publishing, 2002
Paul Gibbon
*Short pulse laser interaction with matter*
Imperial College Press, 2005
David Attwood
*Soft X-rays and Extreme Ultaviolet Radiation*
Cambridge University Press, 1999

Thesis
Thomas Sokollik
http://opus.kobv.de/tuberlin/volltexte/2008/2028/pdf/sokollik_thomas.pdf
Sven Steinke
Andreas Henig
http://edoc.ub.uni-muenchen.de/11483/1/Henig_Andreas.pdf
Rainer Hörlein
http://edoc.ub.uni-muenchen.de/9615/1/Hoerlein_Rainer.pdf
Sebastian Pfotenhauer
http://www.physik.uni-jena.de/qe/Papier/Dissertationen/Diss_Pfotenhauer.pdf
Oliver Jäckel
www.physik.uni-jena.de/inst/polaris/Publikation/Papier/Dissertationen/Diss_Jaeckel.pdf
Thomas Nubbemeyer